RLP-STATSB · RLP Statistics B — Probability Models and Statistical Inference

Unit 2: Estimating with Confidence

Practice Workbook. The same problems, in the same order, as the Practice sections of the online lessons.

Name _____ Period _____

Lesson 2.1 — Estimating a Proportion with Confidence

FLUENCY

1. In a random sample of 1000, 422 said yes. Construct a 90% confidence interval for the population proportion.

2. In a random sample of 800, 223 said yes. Construct a 95% confidence interval for the population proportion.

3. In a random sample of 200, 53 said yes. Construct a 90% confidence interval for the population proportion.

4. In a random sample of 400, 222 said yes. Construct a 95% confidence interval for the population proportion.

5. In a random sample of 500, 419 said yes. Construct a 95% confidence interval for the population proportion.

6. In a random sample of 800, 548 said yes. Construct a 90% confidence interval for the population proportion.

7. Find z* for 90%, 95% and 99% confidence.

8. How large a sample gives a 95% margin of error of ±4 points for a proportion, with no prior estimate?

APPLICATION

9. A random sample of 150 days from Springfield's 2000-2024 daily record (NOAA) contains 55 days with measurable rain. Check the conditions, construct a 95% confidence interval for the proportion of rainy days, and interpret it.

10. Explain what '95% confident' means for the Springfield interval, without using the word 'probability' about the parameter.

11. Before sampling, a planner guesses that about 13% of days have at least 0.25 inch of rain. How many days are needed for a 95% margin of error of ±0.03 for that proportion? Compare with the answer using $p^* = 0.5$.

CHALLENGE

12. A random sample of 40 days contains 3 days with at least 1.5 inches of rain. Why is the usual z-interval not appropriate, and what could you do?

REVIEW

13. X is normal with mean 100 and standard deviation 15. Find $P(90 < X < 119)$.

14. $X \sim \text{Binomial}(n = 7, p = 0.4)$. Find $P(X = 3)$, $P(X \le 3)$, and the mean and standard deviation of X.

Lesson 2.2 — Confidence Intervals for a Mean

FLUENCY

1. A random sample of 15 has mean 66.9 and standard deviation 10.2. Construct a 95% confidence interval for the population mean.

2. A random sample of 12 has mean 52.7 and standard deviation 14.9. Construct a 90% confidence interval for the population mean.

3. A random sample of 25 has mean 57.2 and standard deviation 8.9. Construct a 95% confidence interval for the population mean.

4. A random sample of 12 has mean 72.6 and standard deviation 13.0. Construct a 90% confidence interval for the population mean.

5. A random sample of 20 has mean 67.6 and standard deviation 7.4. Construct a 90% confidence interval for the population mean.

6. A random sample of 30 has mean 59.2 and standard deviation 8.5. Construct a 90% confidence interval for the population mean.

7. Find t* for 95% confidence with n = 10 and with n = 30.

8. Why use t* rather than z* when σ is unknown?

APPLICATION

9. A random sample of 25 days from Springfield's 2000-2024 daily highs (NOAA) has $\bar{x}$ = 67.60°F and s = 25.18°F. Construct and interpret a 95% t-interval for the mean daily high.

10. Springfield daily highs have σ ≈ 18.83°F. How many randomly chosen days would estimate the mean daily high within ±0.5°F with 95% confidence?

11. A random sample of 8 daily rainfall totals at Springfield contains six zeros and two large values. Is a t-interval for the mean appropriate?

CHALLENGE

12. For a sample of 5 with s = 4, compare the 95% margins of error computed with t* and with z*. By what percent does z* understate the margin?

REVIEW

13. (Unit 1) Find the SD of $\bar{x}$ for samples of 49 from a population with $\sigma = 14$.

14. (RLP Statistics A) Name the sampling method: a random sample of days within each month, combined.

Lesson 2.3 — What Confidence Means, and What It Does Not

FLUENCY

1. An ACS estimate has a published margin of error of ±2.4 points (90% confidence). Find the SE and the 95% margin of error.

2. True or false: a 95% confidence interval for mean daily high contains about 95% of individual days' highs.

3. True or false: for the same data, a 99% interval is wider than a 95% interval.

4. Name two sources of error that a survey's margin of error does not include.

5. A poll of 1,068 reports 52% vs 48% with a margin of error of ±3 points. About what is the margin of error for the 4-point lead?

6. Two intervals overlap slightly. Does that prove the two population values are equal?

APPLICATION

7. (Illustrative ACS-style estimates, 90% margins) County A: 14.2% ± 1.8; County B: 11.5% ± 2.1. Is the difference statistically significant at the 90% level?

8. A news story says: 'Support is 47% with a margin of error of 3 points, so fewer than half of voters support the measure.' Evaluate the claim.

9. A small-area ACS estimate reads 8% ± 7 points. What should a careful report say about it?

CHALLENGE

10. Explain why the margin of error for the lead in a single poll is about twice the margin for each candidate's share, when the two shares are near 50%.

REVIEW

11. In a random sample of 500, 373 said yes. Construct a 99% confidence interval for the population proportion.

12. (Lesson 2.2) Why do you round a required sample size up, never down?

Lesson 2.4 — Comparing Two Groups with Confidence Intervals

FLUENCY

1. Paired or independent? Annual temperature at two stations in the same 30 years.

2. Paired or independent? Test scores of 30 students taught with method A and 30 different students taught with method B.

3. Paired or independent? Each volunteer's reaction time with and without a notification sound, in random order.

4. Two independent sample proportions: $\hat{p}_1 = 0.40$ ($n = 200$), $\hat{p}_2 = 0.30$ ($n = 250$). Find the SE of the difference and the 95% interval.

5. Two independent samples: $\bar{x}_1 = 52$, $s_1 = 8$, $n_1 = 40$; $\bar{x}_2 = 47$, $s_2 = 10$, $n_2 = 50$. Find the SE of the difference.

6. Why is a paired interval usually narrower than a two-sample interval on the same data?

APPLICATION

7. Springfield winter days (Dec-Feb) 2000-2024 (NOAA): after a rainy day, rain followed on 234 of 584 days; after a dry day, on 353 of 1,672. Construct a 95% interval for the difference and interpret it.

8. Joplin annual mean temperature (NOAA): 1950-1979 mean 57.39°F, SD 1.39; 1995-2024 mean 58.77°F, SD 1.36 (30 years each). Construct a 95% interval for the difference (recent – early) using technology's df, and write a scope statement.

9. Springfield minus Kansas City precipitation, paired by year, 1995-2024: mean difference 5.339 in, SD 7.204 in. Construct the 95% paired interval. Then compare its SE with the two-sample SE on the same data (1.874 in) and explain the difference.

CHALLENGE

10. In a randomized experiment, the 95% interval for (treatment − control) is (0.4, 4.0) words recalled. In an observational comparison of two eras, the 95% interval is (2.37, 3.68)°F. Write the scope statement for each.

REVIEW

11. A random sample of 15 has mean 42.4 and standard deviation 11.9. Construct a 90% confidence interval for the population mean.

12. (Lesson 2.3) An ACS 90% margin is ±1.5 points. What is the 95% margin?

Unit 2 checkpoint

1. With 66 rainy days in a sample of 200, $\hat{p}$ = _____.

2. Which is a correct interpretation of a 95% confidence level?
   A. 95% of the data lie in the interval
   B. There is a 95% probability μ is in this interval
   C. About 95% of intervals built this way from repeated samples capture the parameter
   D. 95% of samples have the same mean
3. True or false: Doubling the sample size halves the margin of error.
4. For a 95% margin of error of ±3 points with $p^* = 0.5$, n = _____.
5. A 95% interval for a mean with n = 30 and unknown σ should use…
   A. $z^* = 1.96$
   B. $t^* \approx 2.045$
   C. $t^* \approx 1.699$
   D. $z^* = 1.645$
6. True or false: A margin of error covers bias from nonresponse.
7. An ACS margin of ±2.0 points (90%) corresponds to a 95% margin of about _____ points.
8. St. Louis and Springfield temperatures in the same 30 years: which procedure?
   A. Two-sample t
   B. Paired t on differences
   C. One-proportion z
   D. Chi-square
9. An observational interval for a difference between eras excludes 0. What can you conclude?
   A. The cause of the difference
   B. The difference is larger than chance variation would explain; the cause is not identified
   C. Nothing
   D. The data are wrong
10. True or false: If two 95% confidence intervals overlap, the difference between the groups cannot be significant.
11. A sample of 30 daily highs has $\bar{x} = 61.20$ and s = 22.596; the SE is about _____.
12. Which condition is NOT required for a one-proportion z-interval?
    A. Random sample
    B. At least 10 successes and 10 failures
    C. Population is normal
    D. Sample under 10% of population