RLP-STATSB · RLP Statistics B — Probability Models and Statistical Inference

Unit 1: Continuous Distributions and Sampling Distributions

Practice Workbook. The same problems, in the same order, as the Practice sections of the online lessons.

Name _____ Period _____

Lesson 1.1 — Continuous Random Variables and Density Curves

FLUENCY

1. A wait W is uniformly distributed from 0 to 20 minutes. Find the height of the density, P(W > 15), P(5 < W < 8) and the mean wait.

2. For the same uniform wait, find P(W = 7).

3. A waiting time is exponential with mean 5 minutes. Find P(X > 5).

4. For the same exponential wait (mean 5), find P(X < 2).

5. Find the median of an exponential distribution with mean 5.

6. An exponential wait has mean 10 minutes. Find $P(X > 20)$.

7. Can a density curve have height greater than 1 anywhere? Explain.

8. A density curve is symmetric about 12. What is $P(X < 12)$?

APPLICATION

9. (Illustrative) Calls to a school's attendance line arrive at random, on average every 3 minutes. What is the probability of waiting more than 6 minutes for the next call? More than 9 minutes, given that 3 minutes have already passed?

10. A light-rail train arrives exactly every 15 minutes, and you arrive at a random time. Which model fits your wait, uniform or exponential? Find P(wait > 10).

11. For an exponential wait with mean 4 minutes, find $P(2 < X < 6)$ and explain why the mean exceeds the median.

CHALLENGE

12. The density $f(x) = cx$ on $0 \le x \le 2$ (and 0 elsewhere) is a valid density curve for one value of c. Find c, then find $P(X < 1)$.

REVIEW

13. $X \sim$ Binomial($n = 14$, $p = 0.75$). Find $P(X = 12)$, $P(X \le 12)$, and the mean and standard deviation of X.

14. A game pays -5 with probability 1/3; 5 with probability 2/5; 25 with probability 4/15. Find the expected value.

Lesson 1.2 — The Normal Distribution

FLUENCY

1. A distribution is approximately normal with mean 100 and standard deviation 15. Find the z-score of 123.5 and the proportion of values below it.

2. A distribution is approximately normal with mean 3.1 and standard deviation 0.4. Find the z-score of 3.0 and the proportion of values below it.

3. A distribution is approximately normal with mean 64 and standard deviation 2.8. Find the z-score of 61.2 and the proportion of values below it.

4. A distribution is approximately normal with mean 100 and standard deviation 15. Find the z-score of 77.7 and the proportion of values below it.

5. X is normal with mean 170 and standard deviation 9. Find $P(163 < X < 186)$.

6. X is normal with mean 24 and standard deviation 3. Find $P(22 < X < 31)$.

7. X is normal with mean 170 and standard deviation 9. Find $P(154 < X < 180)$.

8. X is normal with mean 100 and standard deviation 15. Find $P(86 < X < 105)$.

9. For a standard normal Z, find $P(-1.96 < Z < 1.96)$ and the 90th percentile of Z.

APPLICATION

10. Springfield annual mean temperature, 1950-2024 (NOAA): mean 56.37°F, SD 1.48°F. Check the 68-95-99.7 rule: 69.3% of the 75 years are within 1 SD and 94.7% within 2 SD. Then use a normal model to estimate the fraction of years at 59°F or above, and compare with the data (5 years).

11. Using N(56.37, 1.48), find the temperature that marks the warmest 10% of Springfield years, and compare with the data's 90th percentile (58.2°F).

12. A normal distribution has 25th percentile 62.0 and 75th percentile 64.4. Find μ and σ.

CHALLENGE

13. A normal distribution has 10% of its values below 50 and 30% above 70. Find μ and σ.

REVIEW

14. Use technology to find the least-squares line and the correlation coefficient for (2, 13), (4, 17), (5, 19), (10, 21), (11, 27), (12, 24).

15. A survey records two yes/no questions. Yes to both: 59; yes to A only: 25; yes to B only: 11; no to both: 8. Find P(A), P(A and B), P(A | B), and decide whether A and B look independent.

Lesson 1.3 — When Is a Normal Model Appropriate?

FLUENCY

1. $X \sim$ Binomial($n = 9$, $p = 0.25$). Find P($X = 8$), P($X \leq 8$), and the mean and standard deviation of X.

2. $X \sim$ Binomial($n = 7$, $p = 0.6$). Find P($X = 2$), P($X \leq 2$), and the mean and standard deviation of X.

3. Check the normal-approximation conditions for Binomial(40, 0.1).

4. Check the conditions for Binomial(200, 0.3).

5. Binomial(100, 0.5): find the mean and SD of the approximating normal distribution.

6. To approximate P(X ≤ 12) for a binomial count with a normal curve, which boundary should you use?

7. To approximate P(X ≥ 25), which boundary should you use?

8. X ~ Binomial(100, 0.3117) (rainy days in a random sample of 100 St. Louis days). Find P(X ≤ 25) exactly and with the normal approximation (continuity correction).

9. X ~ Binomial(50, 0.4). Find P(X ≥ 25) exactly and by the normal approximation.

APPLICATION

10. Springfield calendar-year precipitation, 1950-2024: mean 42.92 in, SD 8.51 in. A normal model gives P(X < 30) ≈ 0.064; the data have 6 of 75 years below 30 inches. The mean (42.92) and median (43.25) are close. Is a normal model reasonable here? Compare with St. Louis snowfall.

11. Springfield annual mean temperature, 1950-2024: 69.3% of years within 1 SD and 94.7% within 2 SD of the mean. The record also has a strong upward trend (see the era comparison in Lesson 3.4). Is a single normal model a good description of these 75 values?

12. A student says: 'n = 30, so the binomial with p = 0.05 is approximately normal.' Evaluate the claim.

CHALLENGE

13. For p = 0.2, what is the smallest n for which both np ≥ 10 and n(1 – p) ≥ 10? Explain why the binomial becomes more normal as n grows.

REVIEW

14. (RLP Statistics A) Why might a binomial model fail for rainy days per week at St. Louis?

15. (RLP Statistics A) Find the z-score of 39.5 in a distribution with mean 18.36 and SD 10.41.

Lesson 1.4 — Sampling Distributions and the Central Limit Theorem

FLUENCY

1. Daily highs at St. Louis (2000-2024) have $\sigma \approx 20.05°F$. Find the SD of $\bar{x}$ for random samples of 25 days, 64 days and 100 days.

2. The population mean daily high is 67.59°F. What is the mean of the sampling distribution of $\bar{x}$ for n = 40?

3. The proportion of St. Louis days at or above 90°F is $p \approx 0.1380$. Find the SD of $\hat{p}$ for n = 50 and n = 200.

4. A population has $\sigma = 12$. What sample size makes the SD of $\bar{x}$ equal to 2?

5. Is the 10% condition met for a sample of 500 days from the 9,132-day St. Louis record?

6. By what factor does the SD of $\bar{x}$ change when n goes from 25 to 100?

7. True or false: the central limit theorem says that a large sample of data will be approximately normal.

APPLICATION

8. Use the CLT to estimate the probability that a random sample of 36 St. Louis days has a mean high above 75°F ($\mu = 67.59$, $\sigma = 20.05$). A simulation of 5,000 such samples gave a proportion of 0.0092; compare.

9. A reporter takes the first 30 days of July at St. Louis as a 'sample' to estimate the station's mean annual daily high. Does the CLT make $\bar{x}$ a good estimate? Explain.

10. Daily rainfall at St. Louis ($\sigma \approx 0.346$ in, extremely skewed) and daily highs ($\sigma \approx 20.1$°F, mildly skewed): for which population is $n = 30$ enough for $\bar{x}$ to be approximately normal? Explain.

CHALLENGE

11. What is the smallest sample size for which the SD of $\bar{x}$ for St. Louis daily highs is at most 1°F?

REVIEW

12. X is normal with mean 500 and standard deviation 100. Find $P(289 < X < 614)$.

13. (Lesson 1.2) For N(56.64, 1.88), find P(X > 59).

14. (RLP Statistics A) A random variable has E(X) = 5 and SD 2. Find the mean and SD of 3X + 1.

Unit 1 checkpoint

1. True or false: For a continuous random variable, P(X = 3) is the height of the density at 3.
2. For a uniform wait from 0 to 12 minutes, P(wait > 9) = _____.
3. For an exponential wait with mean 4, P(X > 6) = e^(−1.5) ≈ _____.
4. A normal model with mean 56.64 and SD 1.88: about what fraction of values exceed 59?
   A. 0.90
   B. 0.104
   C. 0.16
   D. 0.5

5. A normal distribution with 10th percentile 54.2 and 90th percentile 59.0 has mean _____.

6. Which data set is least suited to a normal model?
   A. Annual mean temperature at St. Louis
   B. Annual snowfall at St. Louis
   C. Heights of adult women
   D. Measurement errors

7. True or false: The normal approximation to Binomial(14, 0.5) for $P(X \le 5)$ is much better with a continuity correction.

8. The SD of $\bar{x}$ for samples of 100 from a population with $\sigma = 8.3$ is _____.

9. The mean of the sampling distribution of $\bar{x}$ equals…
   A. $\sigma/\sqrt{n}$
   B. μ
   C. $\bar{x}$ from one sample
   D. 0

10. True or false: The central limit theorem makes a sample of 100 rainfall days approximately normal.

11. For a very skewed population, the sampling distribution of $\bar{x}$ at $n = 30$…
   A. Is always exactly normal
   B. May still be noticeably skewed
   C. Is uniform
   D. Has SD σ

12. True or false: A convenience sample's mean is unbiased if n is large enough.

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