RLP-STATSA · RLP Statistics A — Exploring Data, Study Design and Probability

Unit 4: Probability, Random Variables and the Binomial Model

Practice Workbook. The same problems, in the same order, as the Practice sections of the online lessons.

Name _____ Period _____

Lesson 4.1 — Probability Rules and Two-Way Tables

FLUENCY

1. A survey records two yes/no questions. Yes to both: 33; yes to A only: 37; yes to B only: 51; no to both: 29. Find P(A), P(A and B), P(A | B), and decide whether A and B look independent.

2. A survey records two yes/no questions. Yes to both: 25; yes to A only: 24; yes to B only: 9; no to both: 15. Find P(A), P(A and B), P(A | B), and decide whether A and B look independent.

3. A survey records two yes/no questions. Yes to both: 48; yes to A only: 42; yes to B only: 40; no to both: 41. Find P(A), P(A and B), P(A | B), and decide whether A and B look independent.

4. Two fair dice are rolled. Find P(sum = 5).

5. Two fair dice are rolled. Find P(at least one die shows 3).

6. Two fair dice are rolled. Find P(sum = 3).

7. Two fair dice are rolled. Find P(sum = 12).

8. Events A and B have P(A) = 0.4, P(B) = 0.5 and P(A and B) = 0.2. Find P(A or B) and decide whether A and B are independent.

9. If P(A) = 0.35, what is P(not A)?

APPLICATION

10. St. Louis, January days 2000-2024 (NOAA), paired with the day before: rain yesterday & rain today 101; rain yesterday & dry today 122; dry yesterday & rain today 121; dry yesterday & dry today 430. Find P(rain today), P(rain today | rain yesterday) and P(rain today | dry yesterday). Are the events independent?

11. Kansas City International, winter days (Dec-Feb) 2000-2024: rain-rain 185, rain-dry 310, dry-rain 309, dry-dry 1,452 (yesterday listed first). Compute P(rain yesterday or rain today) with the addition rule, and compare rain persistence at Kansas City with St. Louis's winter value of 0.427.

12. St. Louis summer days (Jun-Aug) 2000-2024, counting only heavy-rain days (PRCP at least 1.00 in): rain-rain 10, rain-dry 83, dry-rain 85, dry-dry 2,122. Find P(heavy rain today). Why should you be cautious about P(heavy rain today | heavy rain yesterday) here?

CHALLENGE

13. Show that if P(A | B) = P(A), then P(B | A) = P(B) (assuming both probabilities are positive).

REVIEW

14. (Unit 3) A study finds that people who own dogs walk more. Is this an experiment or an observational study, and can it show that owning a dog causes more walking?

15. (Unit 2) Suppose a regression of y on x has $r^2 = 0.82$. What does that mean?

Lesson 4.2 — The Multiplication Rule, Trees and Counting

FLUENCY

1. How many different 4-person committees can be chosen from 12 people?
2. In how many ways can 2 different offices be filled from 7 candidates?
3. A group has 13 juniors and 0 seniors. If 4 are chosen at random, find the probability that exactly 4 are juniors.
4. In how many ways can 5 different offices be filled from 13 candidates?
5. In how many ways can 2 different offices be filled from 8 candidates?

6. How many different 3-person committees can be chosen from 10 people?

7. Two cards are drawn without replacement from a standard deck. Find P(both hearts).

8. How many ways can a president, vice-president and secretary be chosen from 12 club members?

APPLICATION

9. (Illustrative rates) A screening test detects a condition 95% of the time when present and gives a false positive 10% of the time when absent. The condition affects 10% of a population. Find P(condition | positive) with a table for 10,000 people.

10. In Powerball (5 white-ball numbers from 1-69 plus a Powerball from 1-26), what is the probability that a single play matches all five white balls but not the Powerball?

11. What is the probability that at least two people in a class of 30 share a birthday (365 equally likely birthdays, ignoring leap days)?

CHALLENGE

12. A club of 28 members randomly chooses 4 to present. Find the probability that you and your best friend are both chosen, and explain the counting.

REVIEW

13. (Lesson 4.1) P(A) = 0.3, P(B) = 0.6, P(A and B) = 0.18. Are A and B independent?

14. (Unit 3) Why does a survey's margin of error not cover nonresponse bias?

Lesson 4.3 — Random Variables, Expected Value and Variance

FLUENCY

1. A game pays −5 with probability 3/11; 10 with probability 4/11; 25 with probability 3/11; 50 with probability 1/11. Find the expected value.

2. A game pays 0 with probability 2/11; 10 with probability 6/11; 40 with probability 1/11; 55 with probability 2/11. Find the expected value.

3. A game pays −10 with probability 5/14; −5 with probability 3/7; 0 with probability 3/14. Find the expected value.

4. A game pays −5 with probability 1/4; 5 with probability 1/8; 40 with probability 5/8. Find the expected value.

5. A game pays 10 with probability 5/17; 20 with probability 4/17; 30 with probability 3/17; 35 with probability 5/17. Find the expected value.

6. Two fair dice are rolled. Find P(sum ≥ 11).

7. Two fair dice are rolled. Find P(at least one die shows 1).

8. A random variable takes values 0, 1, 2 with probabilities 0.5, 0.3, 0.2. Find its mean and standard deviation.

APPLICATION

9. Kansas City International (NOAA), July 1-28 of each year 2000-2024, split into four 7-day weeks (100 weeks). Let Y = the number of days at or above 95°F in a week. Counts for y = 0,…,7: 53, 14, 15, 6, 4, 3, 3, 2. Find the probability distribution, E(Y) and SD(Y).

10. Interpret E(Y) = 1.25 for Kansas City July weeks (95°F or higher), and say what the shape of the distribution adds.

11. A heat-safety program pays for cooling-center hours: \$500 per week plus \$300 for each 95°F day. Using E(Y) and SD(Y) from above, find the mean and SD of the weekly cost $W = 500 + 300Y$.

CHALLENGE

12. For the sum X of two fair dice, compute Var(X) both as $\Sigma(x - \mu)^2 P(x)$ and as $E(X^2) - \mu^2$, and show they agree.

REVIEW

13. (Lesson 4.2) How many 3-person committees can be chosen from 10 people?

14. (Unit 1) Is the Kansas City weekly distribution above better summarized by its mean or its median, and why might neither be 'typical'?

Lesson 4.4 — The Binomial Model

FLUENCY

1. $X \sim \text{Binomial}(n = 7, p = 0.25)$. Find $P(X = 6)$, $P(X \le 6)$, and the mean and standard deviation of X.

2. $X \sim \text{Binomial}(n = 7, p = 0.2)$. Find $P(X = 3)$, $P(X \le 3)$, and the mean and standard deviation of X.

3. $X \sim \text{Binomial}(n = 8, p = 0.5)$. Find $P(X = 6)$, $P(X \le 6)$, and the mean and standard deviation of X.

4. $X \sim \text{Binomial}(n = 17, p = 0.75)$. Find $P(X = 8)$, $P(X \le 8)$, and the mean and standard deviation of X.

5. $X \sim \text{Binomial}(n = 10, p = 0.25)$. Find $P(X = 6)$, $P(X \le 6)$, and the mean and standard deviation of X.

6. $X \sim \text{Binomial}(n = 13, p = 0.2)$. Find $P(X = 10)$, $P(X \le 10)$, and the mean and standard deviation of X.

7. Which condition of the binomial setting fails? Counting how many of the next 7 days are rainy in St. Louis in January.

8. Which condition fails? Drawing 5 cards without replacement from a deck and counting hearts.

APPLICATION

9. (Illustrative) A 70% free-throw shooter takes 10 independent shots. Find $P(X = 10)$, $P(X \geq 8)$, the mean and the SD.

10. For Kansas City July weeks, the fraction of July 1-28 days at or above 95°F is $p = 125/700 \approx 0.179$. A Binomial(7, 0.179) model predicts, for 100 weeks: 25.2, 38.4, 25.0, 9.1, 2.0, 0.3, 0.0, 0.0 weeks with 0,...,7 hot days. The observed counts were 53, 14, 15, 6, 4, 3, 3, 2. Does the binomial model fit? Explain.

11. A quiz has 12 multiple-choice questions with 4 options each. A student guesses on every question. Find the probability of 6 or more correct.

CHALLENGE

12. Show directly that a Binomial(2, p) random variable has mean 2p and variance 2p(1 − p).

REVIEW

13. (Lesson 4.3) A game pays \$10 with probability 0.2 and −\$2 otherwise. Find the expected value.

14. (Unit 2) A line predicts y = 20 + 1.5x. For the point (10, 38), find the residual.

Unit 4 checkpoint

1. If P(A) = 0.5, P(B) = 0.4 and P(A and B) = 0.1, then P(A or B) = _____.
2. P(rain today | rain yesterday) = 0.427 and P(rain today) = 0.306. What does this show?
   A. Independence
   B. Rain on consecutive days is not independent
   C. An error
   D. P(rain) = 0.427
3. True or false: P(A | B) is always equal to P(B | A).
4. Drawing two cards without replacement, P(both aces) = (4/52)(_____/51).
5. How many 5-number combinations can be chosen from 69 numbers?
   A. 69^5
   B. 11,238,513
   C. 1,348,621,560
   D. 345
6. A random variable takes 0 and 10 with probabilities 0.6 and 0.4; its expected value is _____.

7. True or false: An expected value must be one of the variable's possible values.

8. If E(X) = 7 and SD(X) = 2.42, what are the mean and SD of W = 3X − 20?
   A. 1 and 7.25
   B. 1 and −12.7
   C. 21 and 2.42
   D. −13 and 7.25

9. For X ~ Binomial(14, 0.5), P(X ≤ 5) = 3,473/_____.

10. Which is NOT a binomial condition?
    A. Fixed number of trials
    B. Independent trials
    C. Same probability each trial
    D. Outcomes must be equally likely

11. True or false: A binomial model fits the spread of rainy-day counts in St. Louis January weeks, because the means match.

12. A $1 roulette bet on one number (pays $35; 38 pockets) has expected value…
    A. $0
    B. −$0.053
    C. +$0.92
    D. −$1

13. True or false: Choosing names for a county's master jury list by random selection from government records gives every name in those records the same chance of being chosen.

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