RLP-GEOA · RLP Geometry A — Transformations, Congruence, Similarity and Trigonometry

Unit 1: Constructions and Rigid Transformations

Practice Workbook. The same problems, in the same order, as the Practice sections of the online lessons.

Name _____ Period _____

Lesson 1.1 — Definitions and the First Constructions

FLUENCY

1. Name the four undefined notions that geometry in this course starts from.

2. Complete the definition precisely: a circle with center O and radius r is …

3. Define *perpendicular lines* and *parallel lines* precisely.

4. Why can “a round shape” not be used as a definition of a circle in a proof?

5. In the equilateral-triangle construction on a segment AB of length 5 cm, find AC and BC and give the reason for each.

6. A regular hexagon is inscribed in a circle of radius 4 cm by stepping the radius around the circle. Find its perimeter.

7. Joining every other vertex of that hexagon gives an equilateral triangle inscribed in the same circle (radius 4 cm). Find its side length exactly and to the nearest hundredth.

8. A square is inscribed in a circle of radius 5 cm using two perpendicular diameters. Find the side and area of the square.

APPLICATION

9. *(Illustrative map.)* Two schools sit at $A(1, 2)$ and $B(7, 6)$ on a grid in kilometers. A bus hub must be equidistant from both schools and lie on the highway $y = 0$. Where should it go?

10. A hexagonal community garden is inscribed in a circle of radius 6 m. How much fencing does the hexagon need, and how much more would a circular fence need?

11. A designer draws a 12 cm segment, constructs an equilateral triangle on it, and then constructs the perpendicular bisector of the base. How tall is the triangle?

CHALLENGE

12. Explain why the perpendicular-bisector construction of Lesson 1.1 fails if both circles are drawn with a radius less than half of AB.

13. Using only compass and straightedge, describe how to construct a 30° angle, and justify each step.

REVIEW

14. Solve: $3x - 12 = -39$

15. Solve: $-4x - 13 = -21$

16. Find the slope of the line through $(-8, -9)$ and $(-9, -4)$.

17. A right triangle has legs 12 and 6. Find the hypotenuse exactly.

Lesson 1.2 — Construction Lab: Bisectors, Perpendiculars and Parallels

FLUENCY

1. Put these steps for bisecting angle V in order: (a) draw ray VC; (b) draw an arc centered at V crossing the sides at A and B; (c) draw equal arcs centered at A and B meeting at C.

2. A circle centered at P crosses line l at A and B, and then the perpendicular bisector of AB is constructed. What has been constructed?

3. When you copy angle A to point P, which three pairs of equal lengths make triangles ABC and PQR congruent?

4. Two lines are each perpendicular to the same line. What is their relationship?

5. An angle of 70° is bisected. What is each part?

6. A 9 cm segment is bisected with the perpendicular-bisector construction. How long is each part?

7. A square is inscribed in a circle of radius 6 cm. Find the side exactly and the area.

8. In the angle-bisector construction, why must the two arcs centered at A and B have the same radius?

APPLICATION

9. A carpenter needs a cut line through a nail hole P parallel to the straight edge of a board. Describe a compass-and-straightedge method and say why it works.

10. A quilt block is a square inscribed in a circle of radius 10 cm. The four regions between the square and the circle are cut from a contrasting fabric. Find their total area.

11. You fold a paper angle so that its two sides lie on top of each other, then crease. Explain why the crease bisects the angle.

CHALLENGE

12. Describe how to construct a regular octagon inscribed in a circle of radius 10 cm, and find its side length to the nearest hundredth.

REVIEW

13. Solve: $9x + 3 = 21$

14. Solve: $-6x + 9 = -21$

15. Find the slope of the line through $(1, 2)$ and $(-5, -7)$.

16. A right triangle has legs 11 and 5. Find the hypotenuse exactly.

Lesson 1.3 — Rigid Transformations as Functions

FLUENCY

1. Find the image of $(-5, 3)$ under a reflection over the line $y = x$.

2. Find the image of $(1, 3)$ under a reflection over the line $y = x$.

3. Find the image of $(-6, -8)$ under a reflection over the line $y = x$.

4. Find the image of $(-6, 4)$ under a rotation of 90° counterclockwise about the origin.

5. Find the image of $(4, -8)$ under a rotation of 90° counterclockwise about the origin.

6. Find the image of $(-3, -4)$ under a reflection over the line $y = x$.

7. Find the image of $(-1, 5)$ under the translation $(x, y) \rightarrow (x - 5, y - 3)$.

8. Find the image of $(8, -2)$ under a rotation of 180° about the origin.

9. Find the image of $(2, -1)$ under a reflection over the line $y = x$.

10. Find the image of $(0, -4)$ under a rotation of 270° counterclockwise about the origin.

APPLICATION

11. Reflect $(3, -2)$ over the x-axis and then translate by $(x, y) \to (x - 4, y + 1)$. Then do the same two steps in the opposite order. Do you get the same point?

12. Is $(x, y) \to (x + 2, 3y)$ a rigid transformation? Justify with one pair of points.

13. Rotate $(5, 3)$ by $90°$ counterclockwise about the point $(2, 1)$.

14. A reflection over the x-axis followed by a reflection over the y-axis is the same as which single transformation?

CHALLENGE

15. Prove that reflection over the line $y = x$, $(x, y) \to (y, x)$, preserves the distance between any two points.

REVIEW

16. Solve: $-6x - 2 = -8$

17. Solve: $3x + 2 = -16$

18. Find the slope of the line through $(-7, 8)$ and $(7, 0)$.

19. A right triangle has legs 13 and 15. Find the hypotenuse exactly.

Lesson 1.4 — Symmetry in Design: Carrying Figures onto Themselves

FLUENCY

1. Find the image of $(0, -8)$ under the translation $(x, y) \to (x - 4, y + 5)$.

2. Find the image of $(3, -4)$ under a rotation of $180°$ about the origin.

3. Find the image of $(6, -1)$ under a reflection over the y-axis.

4. List every rotation (including $0°$) and count the lines of symmetry of a square.

5. How many lines of symmetry does a rectangle that is not a square have, and what is its non-trivial rotation?

6. What is the smallest positive rotation that carries a regular octagon onto itself?

7. Describe every symmetry of an isosceles trapezoid that is not a rectangle.

8. A parallelogram that is not a rectangle or rhombus: list its symmetries.

9. How many lines of symmetry does a regular pentagon have, and through what points does each pass?

APPLICATION

10. A hubcap has 5 identical spokes and no other markings. Find every rotation (between 0° and 360°) that carries it onto itself.

11. Which capital letters of H, N, S, A and X have 180° rotational symmetry? Which have a vertical line of symmetry?

12. A square tile has a motif *M* in its top-left corner. Specify the single transformation that places a copy in the bottom-right corner turned upside down, and the one that places a mirror-image copy in the top-right corner.

CHALLENGE

13. A regular pentagon is rotated 72° counterclockwise about its center seven times. What single rotation between 0° and 360° has the same effect?

REVIEW

14. Solve: $-8x + 2 = 26$

15. Solve: $-5x - 5 = -30$

16. Find the slope of the line through $(-4, 2)$ and $(-6, -2)$.

17. A right triangle has legs 7 and 2. Find the hypotenuse exactly.

Unit 1 checkpoint

1. Which definition of a circle can be used in a proof?
   A. A round shape
   B. The set of all points in a plane at a given distance from a given point
   C. A polygon with many sides
   D. A curve with no corners
2. True or false: In the equilateral-triangle construction, AC = AB because C lies on the circle centered at A through B.
3. A regular hexagon inscribed in a circle of radius 7 has perimeter _____.
4. Which construction gives a line through P parallel to line l?
   A. Bisect the angle at P
   B. Copy the angle a transversal makes with l to P in the corresponding position
   C. Draw a circle centered at P
   D. Construct the perpendicular bisector of l
5. An angle of 84° is bisected. Each part measures _____°.
6. What is the image of (4, −1) under a rotation of 90° counterclockwise about the origin?
   A. (1, 4)
   B. (−1, −4)
   C. (−4, 1)
   D. (1, −4)
7. What is the image of (−3, 5) under a reflection over the y-axis?
   A. (3, 5)
   B. (−3, −5)
   C. (5, −3)
   D. (3, −5)
8. True or false: The transformation (x, y) → (2x, y) is rigid because it keeps horizontal lines horizontal.
9. A reflection across line m takes P to P′ so that m is the perpendicular _____ of PP′.
10. How many lines of symmetry does a regular hexagon have?
    A. 3
    B. 6
    C. 12
    D. 2
11. True or false: A rectangle that is not a square is symmetric across its diagonals.
12. The smallest positive rotation that carries a regular pentagon onto itself is _____°.
13. Reflect (2, 3) over the x-axis, then translate 4 left. Where does it land?
    A. (−2, −3)
    B. (6, −3)
    C. (−2, 3)
    D. (2, −7)