RLP-ALG2A · RLP Algebra 2A — Sequences, Polynomials, Rational Functions and Complex Numbers

Unit 4: Complex Numbers and Rational Exponents

Practice Workbook. The same problems, in the same order, as the Practice sections of the online lessons.

Name _____ Period _____

Lesson 4.1 — Rational Exponents and Root Functions

FLUENCY

1. Evaluate exactly: $32^{-2/5}$

2. Evaluate exactly: $8^{2/3}$

3. Evaluate exactly: $32^{-3/5}$

4. Evaluate exactly: $100^{3/2}$

5. Evaluate exactly: $128^{3/7}$

6. Evaluate exactly: $64^{-2/3}$

7. Write $\sqrt{x} \cdot \sqrt[3]{x^2}$ as a single power of x.

8. Write $x^{5/3}$ in radical form two ways.

9. State the domain and range of $y = \sqrt{(x - 4)} + 1$ and of $y = \sqrt[3]{(x + 2)}$.

APPLICATION

10. A pendulum's period is $T = 2\pi\sqrt{(L/9.8)}$ seconds. What length gives a 1.5-second period?

11. A cube's surface area in terms of its volume is $S = 6V^{2/3}$. Find S for $V = 64$ cm^3 and for $V = 125$ cm^3.

12. Metabolic rate is roughly proportional to $M^{3/4}$ (Kleiber's rule). If one animal has 16 times the mass of another, how many times the metabolic rate does it have?

CHALLENGE

13. Which is larger, $2^{1/2}$ or $3^{1/3}$? Decide without a calculator.

REVIEW

14. Solve: $4 / (x + 6) = 1 / (x - 1)$

15. Divide $-2x^3 + 6x^2 - x + 8$ by $x - 4$. State the quotient and remainder, and use the remainder to evaluate $p(4)$.

Lesson 4.2 — Solving Radical Equations Lab

FLUENCY

1. Solve $\sqrt{(x + 2)} = x$.

2. Solve $\sqrt{(3x + 1)} = x - 3$.

3. Solve $\sqrt[3]{(x - 5)} = -2$.

4. Solve $\sqrt{(2x - 1)} + 2 = 5$.

5. Solve $\sqrt{(x + 5)} = \sqrt{(2x - 1)}$.

6. Solve $x^{3/2} = 64$.

7. Solve $2\sqrt{x} = x - 3$.

APPLICATION

8. What pendulum length gives a 2.5-second period, using $T = 2\pi\sqrt{(L/9.8)}$?

9. Crash investigators estimate speed from skid length with $v = \sqrt{(30fd)}$ (mph, feet). With $f = 0.7$, how long a skid corresponds to 45 mph?

10. With Kepler's law $T = a^{3/2}$ (years, AU), how far from the Sun is an asteroid with a 5-year period?

CHALLENGE

11. Solve $\sqrt{x} + \sqrt{(x - 5)} = 5$.

REVIEW

12. Solve: $(x + 4) / (x - 1) = 5 / (x - 1)$

13. Multiply: $(-x)(-2x^2 + 5x - 5)$

Lesson 4.3 — Inventing i: The Complex Numbers

FLUENCY

1. Simplify: $(2 - i) + (7 + 4i)$

2. Simplify: $(-7 - i) + (2 + 5i)$

3. Simplify: $(-6 + 6i) + (7 - 6i)$

4. Simplify: $(-7 + 4i)(-1 + 2i)$

5. Simplify: $(-1 - 4i)(-5 - 6i)$

6. Simplify: $(3 - 4i)(-3 + i)$

7. Simplify: i^{12}

8. Simplify: i^{18}

9. Simplify: i^{27}

APPLICATION

10. Compute $(1 + i)^2$ and use it to find $(1 + i)^4$ and $(1 + i)^8$.

11. In circuits, impedance is written as a complex number. Show that $(3 + 4i)(3 - 4i)$ is real and find it.

12. Multiply $2 + 5i$ by i, plot both in the complex plane, and describe the motion.

CHALLENGE

13. Find real numbers a and b with $(a + bi)^2 = -5 + 12i$.

REVIEW

14. Evaluate exactly: $4^{5/2}$

15. Solve: $3 / (x + 2) = 5 / (x + 1)$

Lesson 4.4 — Complex Solutions and the Fundamental Theorem

FLUENCY

1. Solve: $2x^2 - 5x - 8 = 0$

2. Solve: $x^2 + 2x + 4 = 0$

3. Solve: $3x^2 + 3x + 5 = 0$

4. Solve: $x^2 - 5x - 2 = 0$

5. Solve: $3x^2 + 8x + 1 = 0$

6. Solve: $x^2 + 6x - 8 = 0$

7. Write in the form $a + bi$: $(5 + i) / (3 - i)$

8. Write in the form $a + bi$: $(-4 + 2i) / (1 + 3i)$

9. Factor over the complex numbers: $x^2 + 16$ and $x^4 - 81$.

APPLICATION

10. A ball's height is $h(t) = -16t^2 + 40t + 4$ feet. Does it reach 35 feet? Use the discriminant and give its maximum height.

11. Write a quadratic with real coefficients whose zeros are $3 + 2i$ and $3 - 2i$.

12. Solve $(x - 2)^2 = -9$. How many solutions are there, and why must there be that many?

CHALLENGE

13. Find all three complex zeros of $x^3 - 8$.

REVIEW

14. Solve: $3 / (x + 6) = 1 / (x + 4)$

15. Divide $2x^3 - 5x^2 + 2x - 5$ by $x + 4$. State the quotient and remainder, and use the remainder to evaluate $p(-4)$.

Unit 4 checkpoint

1. 8^{2/3} = _____ and 27^{−1/3} = _____.

2. x^{1/2} · x^{1/3} = ?
   A. x^{1/6}
   B. x^{5/6}
   C. x^{2/5}
   D. x

3. True or false: The domain of y = ∛x is x ≥ 0.

4. Solution set of √(x + 7) = x − 5?
   A. {2, 9}
   B. {9}
   C. {2}
   D. No solution

5. True or false: Cube root equations cannot produce extraneous solutions by cubing.

6. (3 + 2i)(1 − 4i) = _____.

7. i²³ = ?
   A. 1
   B. i
   C. −1
   D. −i

8. True or false: √(−4) · √(−9) = 6.

9. Solutions of x² − 4x + 13 = 0?
   A. 2 ± 3i
   B. −2 ± 3i
   C. 2 ± 9i
   D. 4 ± 6i

10. Over the complex numbers, x² + 25 = (x + _____)(x − _____).

11. How many complex zeros, counted with multiplicity, does a degree-2 polynomial have?
    A. 0 to 2
    B. exactly 2
    C. exactly 1
    D. it depends on the discriminant

12. By T = a^{3/2}, a planet with a 27-year period has a = _____ AU.