

Unit 1: Exponential Functions and Logarithms

Practice Workbook. The same problems, in the same order, as the Practice sections of the online lessons.

Name _____ Period _____

Lesson 1.1 — Exponential Growth and Decay, Rewritten

FLUENCY

1. A quantity starts at 8,000 and grows by 2% each year. Write a model and find its value after 15 years, to the nearest whole number.

2. A quantity starts at 1,200 and grows by 2% each year. Write a model and find its value after 5 years, to the nearest whole number.

3. A quantity starts at 2,500 and grows by 3% each year. Write a model and find its value after 12 years, to the nearest whole number.

4. A quantity starts at 2,500 and decreases by 2% each year. Write a model and find its value after 11 years, to the nearest whole number.

5. A quantity starts at 2,500 and decreases by 12% each year. Write a model and find its value after 12 years, to the nearest whole number.

6. A quantity starts at 8,000 and decreases by 6% each year. Write a model and find its value after 7 years, to the nearest whole number.

7. $V(t) = 500(1.06)^t$, t in years. Rewrite to show the monthly growth factor and the per-decade growth factor.

8. A medication (illustrative) has a half-life of 6 hours. What fraction remains after 24 hours, and what is the hourly decay factor?

9. Rewrite $200 \cdot 3^{t/4}$ to show the growth factor per unit of time.

APPLICATION

10. California grew about 0.6% a year in the 2010s. What growth factor per decade is that?

11. Carbon-14 has a half-life of about 5,730 years. What fraction of the original carbon-14 remains in a 3,000-year-old sample?

12. Which is better for a saver: 5% a year compounded yearly, or 0.4% a month compounded monthly? Compare annual factors.

CHALLENGE

13. Which grows faster: a quantity that doubles every 7 years or one that grows 10% a year? Compare annual factors.

REVIEW

14. Simplify: $(-5 - i)(5 - 6i)$

15. Evaluate exactly: $1000^{-2/3}$

Lesson 1.2 — Logarithms and Change of Base Lab

FLUENCY

1. Evaluate: $\log_3 81$

2. Evaluate: $\log_4 256$

3. Evaluate: $\log_2 8$

4. Evaluate: $\log_2 4$

5. Evaluate: $\log_5 625$

6. Evaluate: $\log_{10} (1/100)$

7. Evaluate: $\log_4 64$

8. Evaluate: $\log_4 1$

9. Use change of base to evaluate $\log_3 50$, $\log_6 200$ and $\log_2 0.1$ to four decimal places.

10. Between which two integers is $\log 0.02$? Then evaluate it.

APPLICATION

11. $\text{pH} = -\log[\text{H}^+]$. A sample of Sacramento River water (illustrative) has $[\text{H}^+] = 3.2 \times 10^{-8}$. Find its pH.

12. Sound level in decibels is $10 \log(I/I_0)$. What is the level of a sound 10^8 times the reference intensity? How many times as intense is a 90 dB sound?

13. On the earthquake magnitude scale, each whole step multiplies the measured wave amplitude by 10. The 1989 Loma Prieta earthquake was magnitude 6.9. How many times the amplitude of a magnitude 5.9 quake was it, and of a magnitude 4.9?

CHALLENGE

14. Without a calculator, find the two integers $\log_2 1,000$ lies between, then use change of base to evaluate it.

REVIEW

15. Evaluate exactly: $16^{5/4}$

16. Solve: $x^2 - 7x - 5 = 0$

Lesson 1.3 — Proving the Laws of Logarithms

FLUENCY

1. Write as a single logarithm: $4 \log x + \log y - 2 \log z$

2. Write as a single logarithm: $2 \log x + \log y - 2 \log z$

3. Expand: $\log(x^2y / z^4)$

4. Write as a single logarithm: $4 \log x + \log y - 3 \log z$

5. Expand: $\log(x^4y / z^3)$

6. Write as a single logarithm: $3 \log x + \log y - 3 \log z$

7. Write as a single logarithm: $4 \log x + \log y - 4 \log z$

8. Expand: $\log(x^5y / z^3)$

9. Given $\log 2 \approx 0.3010$ and $\log 3 \approx 0.4771$, find $\log 18$, $\log 0.75$ and $\log 45$.

APPLICATION

10. Doubling the intensity of a sound adds how many decibels? (Level = $10 \log(I/I_0)$.)

11. A solution is 1,000 times as acidic ($[H^+]$ 1,000 times as large) as another. How do their pH values compare? Use a log law.

12. Before calculators, 347×29 was computed as $10^{\log 347 + \log 29}$. Carry this out with logs to four places and compare with the exact product.

CHALLENGE

13. Solve $\log x + \log(x - 3) = 1$.

REVIEW

14. Simplify: i^{13}

15. Divide $-3x^3 + 4x^2 + 7$ by $x + 2$. State the quotient and remainder, and use the remainder to evaluate $p(-2)$.

Lesson 1.4 — Solving Exponential Equations with Logarithms

FLUENCY

1. Solve for x : $2^x = 20$. Give an exact answer using logarithms and a decimal to the nearest thousandth.

2. Solve for x : $10^x = 20$. Give an exact answer using logarithms and a decimal to the nearest thousandth.

3. Solve for x : $2^x = 100$. Give an exact answer using logarithms and a decimal to the nearest thousandth.

4. Solve for x : $3^x = 3.5$. Give an exact answer using logarithms and a decimal to the nearest thousandth.

5. Solve for x : $10^x = 7$. Give an exact answer using logarithms and a decimal to the nearest thousandth.

6. Solve for x : $3^x = 20$. Give an exact answer using logarithms and a decimal to the nearest thousandth.

7. Find the inverse of $f(x) = 5x$.

8. Find the inverse of $f(x) = 6x - 8$.

9. Solve $1,000e^{0.05t} = 2,500$.

10. Find the inverse of $f(x) = 2 \cdot 3^x - 4$ and state its domain.

APPLICATION

11. At 5.5% annual growth, how long does an investment take to double?

12. A medication level is $400(0.8)^t$ mg after t hours. When does it reach 50 mg?

13. With $P(t) = 10,586,223(1.0190)^t$ (California, t years after 1950), in what year does the model reach 50 million?

CHALLENGE

14. Solve $2^x = 3^{x-1}$ exactly and to three decimals.

REVIEW

15. Solve: $6 / (x + 5) = 5 / (x + 3)$

16. Solve: $2x^2 - 7x - 6 = 0$

Unit 1 checkpoint

- Which shows the monthly factor for 1.05^t (t in years)?
 - $(1.05/12)^{12t}$
 - $(1.05^{1/12})^{12t}$
 - $(1.05^{12})^{t/12}$
 - 1.0042^t
- $\log 32 = \underline{\hspace{2cm}}$ and $\log 0.001 = \underline{\hspace{2cm}}$.
- $\log 50 = ?$
 - $\log 7 / \log 50$
 - $\log 50 / \log 7$
 - $\log 50 - \log 7$
 - $\log(50/7)$
- True or false: $\log(M + N) = \log M + \log N$.
- With $\log 2 \approx 0.3010$, $\log 8 \approx \underline{\hspace{2cm}}$.
- In proving $\log_b(MN) = \log_b M + \log_b N$, which exponent law is used?
 - $b^m b^n = b^{m+n}$
 - $(b^m)^n = b^{mn}$
 - $b^0 = 1$
 - $b^{-n} = 1/b^n$
- The solution of $5 \cdot 2^t = 80$ is $t = \underline{\hspace{2cm}}$.
- Solve $3 \cdot 10^{0.2t} = 600$.
 - $t = 5 \log 200$
 - $t = 0.2 \log 200$
 - $t = \log 600 / 0.6$
 - $t = 200 / 0.2$
- True or false: The graph of $y = \log_2 x$ has a vertical asymptote at $x = 0$.
- The inverse of $f(x) = 10^x$ is $f^{-1}(x) = \underline{\hspace{2cm}}$.
- Doubling time at 7% a year is closest to
 - 7 years
 - 10.2 years
 - 14 years
 - 70 years